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Whether one should use null hypothesis testing, confidence intervals, and/or effect sizes is a source of continuing controversy in educational research. An alternative to testing for statistical significance, known as equivalence testing, is little used in educational research. Equivalence testing is useful in situations where the researcher wishes to show that two means are not statistically different. A common equivalence test for comparing the means of two independent samples is reviewed. A simulation study assessed the relationships among effect size, sample size, statistical significance, and statistical equivalence. An example of typical educational research data is reanalyzed using equivalence methodology. A tentative conclusion is drawn about the magnitude of effect size needed to be important. (Contains 8 tables, 7 figures, and 42 references.) (Author/SLD)

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Running head: EQUIVALENCE TESTING WITH HYPOTHESIS TESTING

The Use of Equivalence Testing in Conjunction with Standard Hypothesis Testing and Effect Sizes

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Abstract

Whether one should use null hypothesis testing, confidence intervals, and/or effect sizes is a source of continuing controversy in educational research. An alternative to testing for statistical significance, known as equivalence testing, is little used in educational research. Equivalence testing is useful in situations where the researcher wishes to show that two means are not significantly different. A common equivalence test for comparing the means of two independent samples is reviewed. A simulation study assessed the relationships between effect size, sample size, statistical significance, and statistical equivalence. An example of typical educational research data is reanalyzed using equivalence methodology. A tentative conclusion about the magnitude of effect size needed to be important is drawn.

The Use of Equivalence Testing in Conjunction with Standard Hypothesis Testing and Effect Sizes

The use of statistical inference, particularly via null hypothesis significance testing, is an extremely common but contentious practice in educational research. Both the pros and the cons of hypothesis testing have been argued in the literature for several decades. Some support the continued usage of significance testing (Abelson, 1997; Hagan, 1997; Harris, 1997; McLean & Ernest, 1998), others desire a greater reliance on alternatives such as confidence intervals or effect sizes (Cohen, 1992, 1994; Knapp, 1998; Thompson, 1998a, 1998b; Vacha-Haase, 2001), and still others advocate an outright ban on significance testing (Carver, 1993; Nix & Barnette, 1998; Schmidt & Hunter, 1997). The references included here are by no means close to being an exhaustive list. This debate is not limited to our research community; for instance, it is also being argued in ecology (McBride, 1999; Anderson, Burnham, & Thompson, 2000). Many in the statistical community outside of our niche of educational and psychological research, though, are either unaware of this debate or feel that it is trivial (Krantz, 1999).

The objective of this paper is not to continue this heated argument, but rather to borrow the method of equivalence testing from biostatistics, as suggested by Bartko (1991), and using it in conjunction with standard hypothesis testing in educational research. Lehmann (1959) anticated the need for interval testing in his classic volume on the theory of hypothesis testing. Many of the currently employed methods of equivalence testing were developed in the 1970's and 1980's to address biostatistical and pharmaceutical problems (Westlake, 1976, 1979; Schuirmann, 1981; Anderson & Hauck, 1983; Patel & Gupta, 1984; Schuirmann, 1987). Rogers, Howard, and Vessey (1993) introduced the use of equivalence testing methods to the social sciences. Serlin (1993) essentially suggested equivalence testing when he suggested the use of "range", rather than "point", null hypotheses.

Background

Standard null hypothesis significance testing dates back to the pioneering theoretical work of Fisher, Neyman, and Pearson. Hypothesis testing can be found in almost every textbook of statistical methods and thus will not be further elaborated on here. Equivalence testing, on the other hand, is a newer technique and one that is unfamiliar to most researchers in education and the social sciences.

Equivalence testing was developed in biostatistics to address the situation where the goal is not to show that the mean of one group is greater than the mean of another group (i.e. the superiority of one treatment to another), but rather to establish that two methods are equal to one another. A common application of this idea in biostatistics is to show that a less expensive "generic" medication is as effective as the more expensive "brand-name" medication. In equivalence testing, the null hypothesis is that the two groups are not equivalent to one another, and hence rejection of the null indicates that the two groups are equivalent. This differs from standard significance testing where the null hypothesis states that the group means are equal and rejection of the null indicates that the two groups are statistically different. A common methodological mistake in research is to conclude that the null hypothesis is true (i.e. two groups have equal means) based on the failure to reject it. This action fails to recognize that the failure to reject H_0 is often merely a Type II error, especially when the sample sizes are small and the power of the test is low.

An explanation of the theory of equivalence testing can be found in Berger and Hsu (1996). Here, we will merely review the most commonly implemented method used for establishing the equivalence of two population means for an additive model, where the difference of means is considered. The multiplicative model, which looks at the ratio of means, will not be considered further in this paper. The commonly used procedure in biostatistics for this problem is to use the "two one-sided tests" procedure, or

TOST (Westlake, 1976, 1979; Schuirmann, 1981, 1987). With the TOST, the researcher will consider two groups equivalent if he can show that they differ by less than some constant τ, the equivalence bound in both directions. The constant τ is often chosen to be a percentage (such as 10% or 20%) of the mean of the control group, although τ can also be chosen to be the smallest absolute difference between two means that is large enough to be practically important.

The null hypothesis (i.e. the means are different) for the TOST is

$$H_0 : |\mu_1 - \mu_2| \geq \tau$$

or

$$H_0 : \mu_1 - \mu_2 \geq \tau \quad \text{or} \quad \mu_1 - \mu_2 \leq -\tau$$

The alternative hypothesis (i.e. the means are equivalent) is

$$H_1 : |\mu_1 - \mu_2| < \tau$$

or

$$H_1 : -\tau < \mu_1 - \mu_2 < \tau$$

The first one-sided test seeks to reject the null hypothesis that the difference between two means is less than or equal to $-\tau$; similarly, the second one-sided test seeks to reject the null hypothesis that the difference in the means is greater than or equal to τ. If the one-sided test with the larger p-value leads to rejection, then the two groups are considered to be equivalent.

For the first one-sided test, we compute the test statistic

$$t_1 = \frac{\bar{x}_1 - \bar{x}_2 + \tau\bar{x}_2}{s_p\sqrt{1/n_1 + 1/n_2}}$$

where s_p is the pooled standard deviation of the two samples and compute the p-value as

$$p_1 = P(t_\nu > t_1)$$

where t_ν is a random variable from the t distribution with $\nu = n_1 + n_2 - 2$ degrees of freedom.

The second one-sided test is similar to the first. The test statistic is

$$t_2 = \frac{\bar{x_1} - \bar{x_2} - \tau\bar{x_2}}{s_p\sqrt{1/n_1 + 1/n_2}}$$

and the p-value is

$$p_2 = P(t_\nu < t_2)$$

If we let $p = \max(p_1, p_2)$, then the null hypothesis of nonequivalence is rejected if $p < \alpha$.

The choice of τ is a difficult choice that is up to the researcher. This choice is analogous to the selection of an appropriate alpha level in standard significance testing, an appropriate level of confidence in interval estimation, or a sufficiently large effect size, and should be made carefully. Knowledge of the situation at hand should be used to specify the maximum difference between population means that would be considered clinically trivial. Researchers in biostatistics typically have the choice made for them by government regulation.

As in standard hypothesis testing, an equivalency confidence interval can also be constructed. If the entire confidence interval is within $(-\tau, \tau)$, then equivalence between the groups is indicated. If the entire confidence interval is within either $(-\tau, 0)$ or $(0, \tau)$ (i.e. zero is not in the interval), then we would reject the null hypotheses of both a significance and and equivalence test. In that case, we could make the somewhat discomforting conclusion that the difference of means was both statistically significant and equivalent.

It is important to note that the equivalency confidence interval is expressed at the $100(1 - 2\alpha)\%$ level of confidence. Rogers et al. (1993) noted that if one performs both a standard significance test and an equivalence test on the same data set, making either a "reject" or "fail to reject" decision, that there are four possibilities. These four conditions are given in Table 1.

Insert Table 1 about here

The second condition "equivalent and different", a simultaneous rejection of both inferential procedures, could happen in a situation where large samples provide "too much power", resulting in a trivial difference in means being statistically significant. The equivalence test (and the effect size) should detect the small magnitude of these mean differences. The fourth condition indicates that there is insufficient evidence to conclude that the groups are either equivalent or different. This would most likely occur when the samples are very small and/or the group variances are very large.

Effect Size Measures for Difference of Means

The effect size for the difference of means is the standardized difference between the groups (Fan, 2001). We will use the parameter

$$\delta = \frac{\mu_1 - \mu_2}{\sigma}$$

to represent the effect size of the population, where μ_1 and μ_2 are the population means and σ^2 is the common variance.

Of course, δ is typically unknown and needs to be estimated. Cohen's d (1988) is a statistic often used for this purpose. The effect size (ES) is found with

$$d = \frac{\bar{x}_1 - \bar{x}_2}{s_{pooled}}$$

where

$$s_{pooled} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

is the pooled standard deviation of the two samples. We stress that Cohen's d is a sample statistic and that d has a sampling distribution like other estimates (e.g. $\bar{x}$). Also,

Cohen's d is biased for δ (i.e. $E(d) \neq \delta$). A modification due to Hedges (1981, 1982) is unbiased for δ.

Cohen (1988) gave some suggestions for interpreting d. An effect size of $d = 0.20$ is deemed "small", $d = 0.50$ is "medium", and $d = 0.80$ is "large". It is becoming, rather regrettably in our opinion, common for researchers to rigidly apply Cohen's suggestions. Absolute reliance on Cohen's rule of thumb is as misguided as blind adherence to a particular level of significance (e.g. $\alpha = 0.05$). As Thompson (2001) said, "we would merely be being stupid in another metric".

Typical Example of Educational Data

Rogers et al. (1993) provided empirical examples of the application of equivalence testing on data from the psychological literature. Here, we will do the same with an example from the educational research literature. This will demonstrate that there often exist situations where a statistically significant difference between groups coincides with the groups being statistically equivalent. This is the "equivalent and different" condition that is typically associated with a small to moderate effect size, as opposed to the strong effect sizes that typically occur with the "different" condition and the weak effect sizes that occur with the "equivalent" condition.

Benson (1989), in a study concerning statistical test anxiety, presented means and variances for a sample of 94 males and 123 females on seven variables. Using standard hypothesis testing methods (i.e. t-tests), significant group differences were found for: prior math courses, math self-concept, self-efficacy, and statistical test anxiety. However, after calculating Cohen's d,

$$d = \frac{\bar{x_1} - \bar{x_2}}{s_p}$$

as an effect size (ES) measure and the use of the TOST equivalence test, we see that only prior math courses and statistical test anxiety are "different" between males and

females. Not surprisingly, the two largest effect sizes are found for these two variables. Table 2 shows results of both traditional significance and equivalence tests for the Benson data.

Statistical significance was defined as a rejection of H_0 with $\alpha = 0.05$ and equivalence was defined as a rejection of H_0 with $\alpha = 0.10$. The reason for the two different significance levels is because while a traditional significance test at level α corresponds to a $100(1-\alpha)\%$ confidence interval, an equivalence test at level α corresponds to a $100(1-2\alpha)\%$ equivalence interval. We selected $\tau = 0.2$ (i.e. 20% of the mean of the female group). This choice was arbitrary and by no means should be taken as a choice recommended for all equivalence problems. The results could differ with different choices for τ.

Insert Table 2 about here

The Power of Significance and Equivalence Tests

The power function for the independent samples t-test is well known. For a test of statistical significance, power is the probability of rejecting the null hypothesis that the population means are equal when they are in fact not equal. Assuming equal variances, the power K_{sig} for the two-sided alternative $H_A : \mu_1 - \mu_2 \neq 0$ is given by (SAS Institute, 1998; O'Brien & Lohr, 1984):

$$K_{sig} = P(t < t_{1-\alpha/2,\nu}, \nu, NC) + 1 - P(t < t_{\alpha/2,\nu}, \nu, NC)$$

where $\nu = n_1 + n_2 - 2$ are the degrees of freedom, $t_{\cdot,\nu}$ is the $\cdot$ quantile of the (central) t-distribution with ν df, and $NC = (\frac{\mu_1 - \mu_2}{\sigma})/\sqrt{\frac{2}{n}}$ is the noncentrality parameter.

The power of an equivalence test is the probability of rejecting that the means are different by at least some equivalence bound τ when the means are in fact equivalent (i.e.

differ by less than τ). The computation of this power, K_{equiv}, requires use of the bivariate non-central t-distribution (Phillips, 1990):

$$K_{equiv} = P(t_1 \geq t_{1-\alpha,\nu} \quad \text{and} \quad t_2 \leq -t_{1-\alpha,\nu})$$

where (t_1, t_2) has a bivariate non-central t-distribution. Owen (1965) showed that probabilities from the bivariate non-central t-dsitribution could be computed as the difference of two definite integrals that are known as Owen's Q functions.

Statistical software packages, such as SAS/GRAPH (SAS Institute, 1998), can be used to graph the power functions of both the tests of statistical significance and equivalence. Figure 1 shows the power of the independent samples t-test and the TOST for various effect sizes with a sample size of $n = 200$ per group and an equivalence bound of $\tau = 0.2$. Figure 2 is a similar graph, but with $\tau = 0.4$.

Insert Figure 1 about here

Insert Figure 2 about here

Figure 3 considers the power of the independent samples t-test and the TOST for samples sizes per group ranging from 10 to 500 with fixed effect size $\delta = 0.2$ and fixed equivalence bound $\tau = 0.2$. Figure 4 makes a similar comparison, with $\delta = \tau = 0.4$. Figure 5 uses $\delta = 0.2$ and $\tau = 0.4$.

Insert Figure 3 about here

Insert Figure 4 about here

Insert Figure 5 about here

Simulation Study

Of interest to us is the probability of rejecting both the null hypotheses (of non-significance and non-equivalence) simultaneously. We designed a small simulation study to assess the power of simultaneously concluding that two means are both statistically different and equivalent.

As is always the case with Monte Carlo studies, the choices of simulation parameters are difficult to make and are somewhat arbitrary. We endeavored to simulate situations that were likely to be encountered in actual quantitative data analysis. We also made some simplifying assumptions to keep the number of simulations and associated tables and figures to a reasonable level.

We assumed that both of our populations were always normally distributed with a common variance $\sigma^2 = 1$. Six different sample sizes per group ($n = 10, 20, 50, 100, 200, 500$) were chosen; only equally sized groups were used in this study. Six different values for the effect size parameter ($\delta = 0, 0.1, 0.2, 0.3, 0.4, 0.5$) were used, reflecting situations from no effect (i.e. equivalent population means) to a "medium" effect size (i.e. population means that differ by one half of a standard deviation). Three different equivalence bounds ($\tau = 0.1, 0.2, 0.4$) were used, defining the minimum difference between means that is practically important (i.e. non-equivalent) to be either 10%, 20% or 40% of μ_1.

Hence, we have a fully crossed design with $6 \times 6 \times 3 = 108$ cells. Within each cell (i.e. combination of sample size, effect size, and equivalence bound) , 10000 simulations were run. The R statistical computing environment was used to conduct the simulations. Each simulation consisted of generating n random normal variates with mean $0 + \delta$ and variance 1 and a second, independent set of n random normal variates with mean 0 and variance 1. The independent samples t-test and the TOST with equivalence bound τ was conducted for each simulation, and the number of rejections of each test, along with the number of simultaneous rejections of both procedures and the number of failures to reject either procedure, were noted.

Tables 3 through 8 show the number of rejections of the null hypotheses of the equivalence test, both tests, the significance test, and neither test. Columns involving the equivalence test are in *italics*; columns involving the significance test are in **bold-face**. Note that the power of the equivalence test for each situation can be found by dividing the sum of the italicized columns by 10000. Similarly, the power of the significance test is obtained by dividing the sum of the columns in bold-face by 10000.

Insert Table 3 about here

Insert Table 4 about here

Insert Table 5 about here

Insert Table 6 about here

Insert Table 7 about here

Insert Table 8 about here

The results of the simulation study were used to obtain graphs of the approximate power of rejecting both tests simultaneously, for sample sizes ranging from 0 to 500 and equivalence bounds $0.1 \leq \tau \leq 0.5$. The graphs were generated with the SAS/GRAPH software package, utilizing the G3GRID and G3D procedures. The G3GRID procedure interpolated, using the default method of bivariate interpolation (Akira, 1978; SAS Institute, 1998), the power of simultaneous rejection for combinations of n and τ that were not included in the simulation design. The G3D procedure then produced a smoothed three-dimensional surface graph of the interpolated data set. Figure 6 is the power of simultaneous rejection with true effect size $\delta = 0$. Figures 7 is a similar graph for a "small" true effect size $\delta = 0.2$.

Insert Figure 6 about here

Insert Figure 7 about here

Discussion

The data originally collected and analyzed with traditional significance tests by (Benson, 1989) showed a statistically significant difference between the means of male and female statistics students on six variables (GPA, number of prior math courses, math self-concept, self-efficacy, general test anxiety, and statistical test anxiety) and failed to find a significance for only one variable (achievement). We computed Cohen's d as an effect size. Not surprisingly, the smallest absolute effect size of 0.04 was found for the non-significant variable, while the absolute effect sizes of the six significant variables ranged from 0.24 to 0.66.

We then re-analyzed Benson's data using the TOST procedure for testing for statistical equivalence. This analysis showed that only two variables, number of prior math courses and statistical test anxiety, were "different" (i.e. significant and not equivalent). Not coincidentally, these were the two variables with the strongest absolute effect sizes of 0.60 and 0.66. The non-significant variable (achievement) was found to be statistically equivalent, and the absolute effect size was virtually zero. Four of the variables (GPA, math self-concept, self-efficacy, and general test anxiety) yielded conflicting results of "equivalent and different" since they rejected the null hypotheses of both the statistical and equivalence tests. It is likely that the difference in the means of these four variables, while statistically significant, is trivial. The absolute effect sizes of these four variables ranged from 0.24 to 0.51. This encompasses a range of effect sizes that is often classified as "small" to "medium" Cohen (1988), notwithstanding Lenth's (2001) warnings against using "canned" effect sizes.

We noticed that whenever the effect size δ is less than the equivalence bound τ, then the power of the equivalence test was approaching unity as n increased. This convergence was slow when δ was nearly equal to τ. Essentially, if the effect size parameter is less than the minimum difference that the researcher considers to be

practically important (i.e. the minimum difference between means large enough to matter), we will reject the null of the TOST and conclude equivalence with power increasing to unity with larger sample sizes.

If $\delta > \tau$, the the power of the significance test approaches unity and the power of the equivalence test approaches zero as the sample size n increases. This is the situation where the effect size parameter exceeds the specified maximum for practical importance; we will reject the t-test and conclude statistical significance with power increasing to unity as the sample size increases.

When $\delta = \tau$, then the power of the equivalence test will approach twice the nominal alpha level (e.g. $2\alpha = 2 \times 0.05 = 0.10$). This occures because the effect size parameter happens to coincide with the specified equivalence bound. Rejecting the TOST (i.e. concluding equivalence) is a type I error, made with probability 2α. The probability is twice the nominal α since an equivalence test at level α corresponds to a $100(1 - 2\alpha\%$ equivalence interval.

When $0 < \delta < \tau$, then the power of both the significance and equivalence tests approaches unity (often slowly) as n increases. This is the situation where the null hypothesis of a significance test is false (i.e. the difference of means is not equal to zero), but the true difference is too small to be considered practically significant, where τ is the minimum difference between means that is considered "important".

It appears to be somewhat common with "real" data to have situations where the tests of statistical significance and equivalence are simultaneously rejected for reasonable choices of significance level α and equivalence bound τ. Our re-analysis of the Benson (1989) data yielded 4 simultaneous rejections out of 7 variables. Rogers et al. (1993) obtained 1 simultaneous rejection out of 13 when re-analyzing data from Cannon, Bell, Fowler, Penk, and Finkelstein (1990) concerning MMPI scores for alcohol versus drug-dependent subjects and 1 simultaneous rejection out of 27 from the study of Zabin,

Hirsch, and Emerson (1990) concerning differences between pregnant adolescent females who elect abortion versus those who carry the baby to term.

The simulated power of simultaneous rejection found in our limited simulation study, as shown in Figures 6 and 7, showed that the probability of simultaneous rejection was low when the assumptions of the inferential tests (i.e. normality, equal variances, equal sample sizes between groups) was low except when both n and τ were large. It is possible that "simultaneous rejection" will be more likely with real data than (at least our) simulated data because real data will surely violate the normality and homoscedasticity assumptions. We speculate simultaneous rejection will be more common, and thus potentially more problematic for the researcher using equivalence testing in conjunction with standard hypothesis testing, when the data is non-normal and heteroscedastic.

We find the magnitude of effect sizes obtained from the statistical re-analysis of typical educational research data to be troubling. Benson's data was of a decent size (groups of 94 and 123 subjects), but an effect size as large as 0.51 yielded both statistical significance (rejecting that the male mean was equal to the female mean) and equivalence (rejecting that the absolute difference of the male and female means were within a constant τ). We make the tentative conjecture that the effect size conventions of Cohen (i.e. 0.2 is small, 0.5 is medium, 0.8 is large) might not be large enough. It is even possible that making any recommendation about the desired magnitude of an effect size independent of the sample sizes and variablity of the populations might be futile (Lenth, 2001).

It would be desirable to extend the simulation study to consider several scenarios ignored here. In particular, more attention needs to be given to situations where one of more of the following conditions are true:

1. the populations are non-normal;
2. the variances are not equal;

3. the sample sizes of the groups are not equal.

It would also be desirable to analtyically determine the power function for simultaneous rejection of the significance and equivalence tests, if possible. We will continue to strive for a greater understanding of the link between the effect size and the results of the significance and equivalence tests.

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Table 1

Possible Combinations of Significance and Equivalence Testing

Significance Test	Equivalence Test	Term
Fail to Reject	Reject	Equivalent
Reject	Reject	Equivalent and Different
Reject	Fail to Reject	Different
Fail to Reject	Fail to Reject	Equivocal

Table 2

Comparing Significance and Equivalence Testing for the Berson Data

	Males ($N = 94$)		Females ($N = 123$)		Effect	Sig.	Equiv.	
Variable	M	SD	M	SD	Size	p-value	p-value	Category
GPA	3.05	0.44	3.16	0.47	-0.24	0.040	< 0.001	Equiv. & Diff.
Prior Math Courses	3.45	2.14	2.20	2.01	0.60	<0.001	0.998	Different
Math Self-concept	25.77	5.96	23.20	7.05	0.39	0.002	0.012	Equiv. & Diff.
Self-efficacy	12.68	1.77	11.62	2.30	0.51	<0.001	<0.001	Equiv. & Diff.
General Test Anxiety	36.38	10.49	40.62	12.25	-0.37	0.004	0.007	Equiv. & Diff.
Achievement	32.56	5.68	32.26	7.55	0.04	0.374	<0.001	Equivalent
Statistical Test Anxiety	32.65	12.57	41.84	14.83	-0.66	<0.001	0.663	Different

Table 3

Simulated Power of the Tests of Statistical Equivalence and Significance, Effect Size $\delta = 0$

Equivalence Bound τ	Sample Size (per group) N	Number of Rejections (10000 Simulations)			
		Equivalent	***Both***	**Different**	Neither
0.1	10	*0*	***0***	**506**	9494
	20	*0*	***0***	**500**	9500
	50	*0*	***0***	**476**	9524
	100	*0*	***0***	**535**	9465
	200	*0*	***0***	**504**	9496
	500	*2337*	***0***	**511**	7152
0.2	10	*0*	***0***	**496**	9504
	20	*0*	***0***	**507**	9493
	50	*0*	***0***	**485**	9515
	100	*1063*	***0***	**546**	8391
	200	*5121*	***0***	**514**	4365
	500	*9386*	***3***	**490**	121
0.4	10	*10*	***0***	**486**	9504
	20	*370*	***0***	**469**	9161
	50	*5279*	***0***	**481**	4240
	100	*8757*	***0***	**457**	786
	200	*9493*	***444***	**63**	0
	500	*9483*	***517***	**0**	0

Table 4

Simulated Power of the Tests of Statistical Equivalence and Significance, Effect Size $\delta = 0.1$

Equivalence Bound τ	Sample Size (per group) N	Number of Rejections (10000 Simulations)			
		Equivalent	***Both***	**Different**	Neither
0.1	10	*0*	***0***	**535**	9465
	20	*0*	***0***	**606**	9394
	50	*0*	***0***	**817**	9183
	100	*0*	***0***	**1118**	8882
	200	*0*	***0***	**1652**	8348
	500	*0*	***709***	**3366**	5925
0.2	10	*0*	***0***	**521**	9479
	20	*0*	***0***	**605**	9395
	50	*1*	***0***	**786**	9213
	100	*793*	***0***	**1090**	8117
	200	*3452*	***0***	**1687**	4861
	500	*6192*	***15***	**3486**	307
0.4	10	*11*	***0***	**565**	9424
	20	*347*	***0***	**622**	9031
	50	*4759*	***0***	**772**	4469
	100	*7902*	***0***	**1044**	1054
	200	*8361*	***1196***	**443**	0
	500	*6521*	***3475***	**4**	0

Table 5

Simulated Power of the Tests of Statistical Equivalence and Significance, Effect Size $\delta = 0.2$

Equivalence Bound τ	Sample Size (per group) N	Number of Rejections (10000 Simulations)			
		Equivalent	***Both***	**Different**	Neither
0.1	10	*0*	***0***	**727**	9273
	20	*0*	***0***	**962**	9038
	50	*0*	***0***	**1727**	8273
	100	*0*	***0***	**2865**	7135
	200	*0*	***0***	**5193**	4807
	500	*16*	***0***	**8880**	1104
0.2	10	*0*	***0***	**699**	9301
	20	*0*	***0***	**950**	9050
	50	*0*	***0***	**1678**	8322
	100	*408*	***0***	**2908**	6684
	200	*951*	***0***	**5207**	3842
	500	*915*	***7***	**8924**	154
0.4	10	*8*	***0***	**734**	9258
	20	*296*	***0***	**967**	8737
	50	*3397*	***0***	**1677**	4926
	100	*5485*	***0***	**2890**	1625
	200	*4886*	***2800***	**2314**	0
	500	*1167*	***8534***	**299**	0

Table 6

Simulated Power of the Tests of Statistical Equivalence and Significance, Effect Size $\delta = 0.3$

Equivalence Bound τ	Sample Size (per group) N	Number of Rejections (10000 Simulations)			
		Equivalent	***Both***	**Different**	Neither
0.1	10	*0*	***0***	**947**	9053
	20	*0*	***0***	**1540**	8460
	50	*0*	***0***	**3144**	6856
	100	*0*	***0***	**5594**	4406
	200	*0*	***0***	**8482**	1518
	500	*0*	***0***	**9973**	27
0.2	10	*0*	***0***	**985**	9015
	20	*0*	***0***	**1501**	8499
	50	*0*	***0***	**3203**	6797
	100	*104*	***0***	**5681**	4215
	200	*95*	***0***	**8524**	1381
	500	*19*	***1***	**9973**	7
0.4	10	*11*	***0***	**991**	8998
	20	*225*	***0***	**1563**	8212
	50	*2061*	***0***	**3133**	4806
	100	*2796*	***0***	**5602**	1602
	200	*1516*	***2374***	**6110**	2167
	500	*23*	***6115***	**3862**	0

Table 7

Simulated Power of the Tests of Statistical Equivalence and Significance, Effect Size $\delta = 0.4$

Equivalence Bound τ	Sample Size (per group) N	Number of Rejections (10000 Simulations)			
		Equivalent	***Both***	**Different**	Neither
0.1	10	*0*	***0***	**1335**	8665
	20	*0*	***0***	**2333**	7667
	50	*0*	***0***	**5015**	4985
	100	*0*	***0***	**8069**	1931
	200	*0*	***0***	**9769**	231
	500	*0*	***0***	**10000**	0
0.2	10	*0*	***0***	**1344**	8656
	20	*0*	***0***	**2341**	7659
	50	*0*	***0***	**5077**	4923
	100	*23*	***0***	**8110**	1867
	200	*1*	***0***	**9784**	215
	500	*0*	***0***	**10000**	0
0.4	10	*9*	***0***	**1402**	8589
	20	*164*	***0***	**2346**	7490
	50	*933*	***0***	**5099**	3968
	100	*932*	***0***	**8075**	993
	200	*232*	***806***	**8962**	0
	500	*0*	***1025***	**8975**	0

Table 8

Simulated Power of the Tests of Statistical Equivalence and Significance, Effect Size $\delta = 0.5$

Equivalence Bound τ	Sample Size (per group) N	Number of Rejections (10000 Simulations)			
		Equivalent	***Both***	**Different**	Neither
0.1	10	*0*	***0***	**1897**	8103
	20	*0*	***0***	**3383**	6617
	50	*0*	***0***	**6981**	3019
	100	*0*	***0***	**9428**	572
	200	*0*	***0***	**9985**	15
	500	*0*	***0***	**10000**	0
0.2	10	*0*	***0***	**1804**	8196
	20	*0*	***0***	**3437**	6563
	50	*0*	***0***	**6905**	3095
	100	*1*	***0***	**9429**	570
	200	*0*	***0***	**9987**	13
	500	*0*	***0***	**10000**	0
0.4	10	*7*	***0***	**1866**	8127
	20	*117*	***0***	**3425**	6458
	50	*370*	***0***	**6938**	2692
	100	*236*	***0***	**9378**	386
	200	*13*	***108***	**9879**	0
	500	*0*	***28***	**9972**	0

Figure Captions

Figure 1. Power of the Tests of Significance and Equivalence with $N = 200$ and $\tau = 0.2$.

Figure 2. Power of the Tests of Significance and Equivalence with $N = 200$ and $\tau = 0.4$.

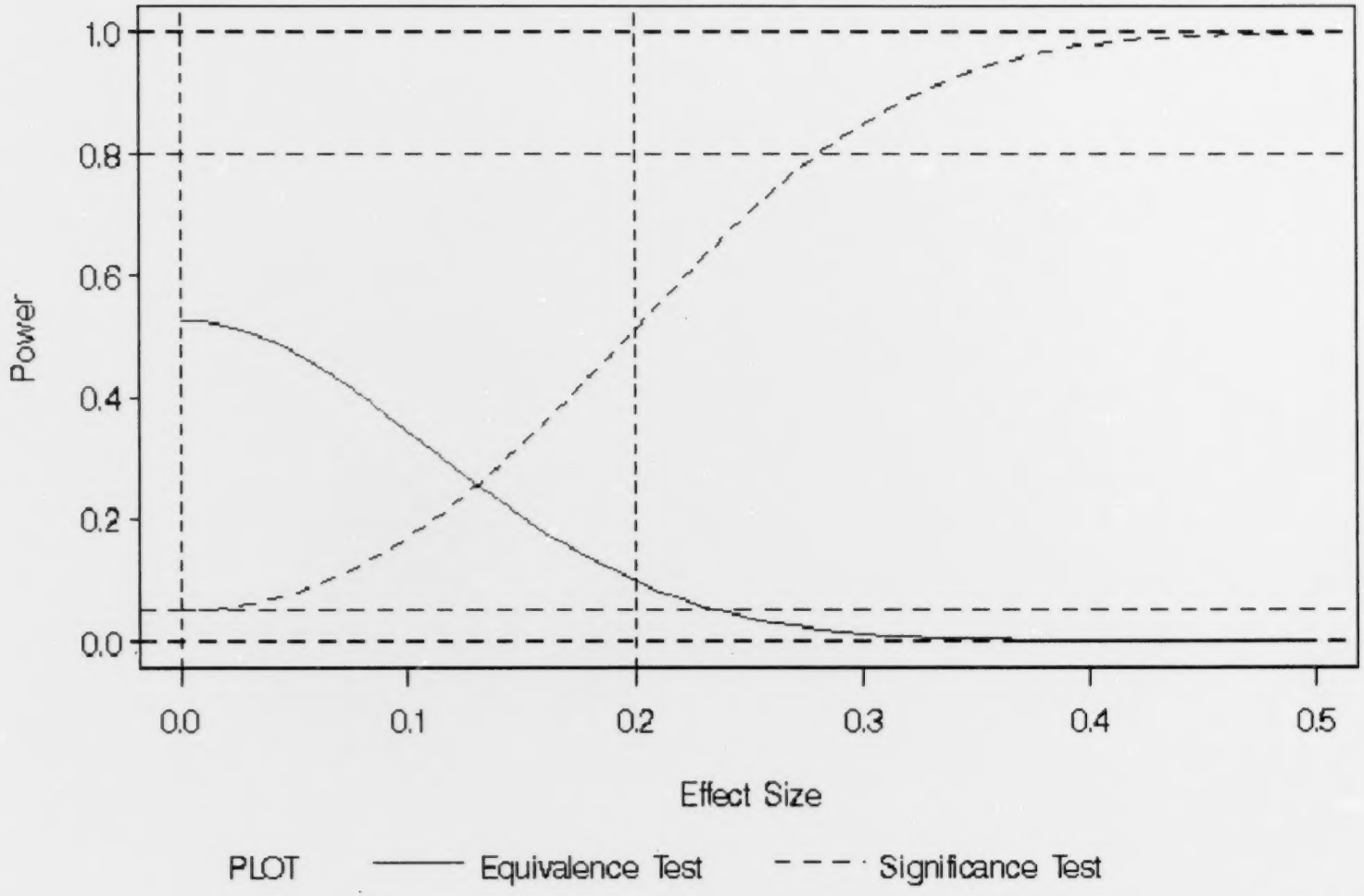
Figure 3. Power of the Tests of Significance and Equivalence with $\delta = \tau = 0.2$.

Figure 4. Power of the Tests of Significance and Equivalence with $\delta = \tau = 0.4$.

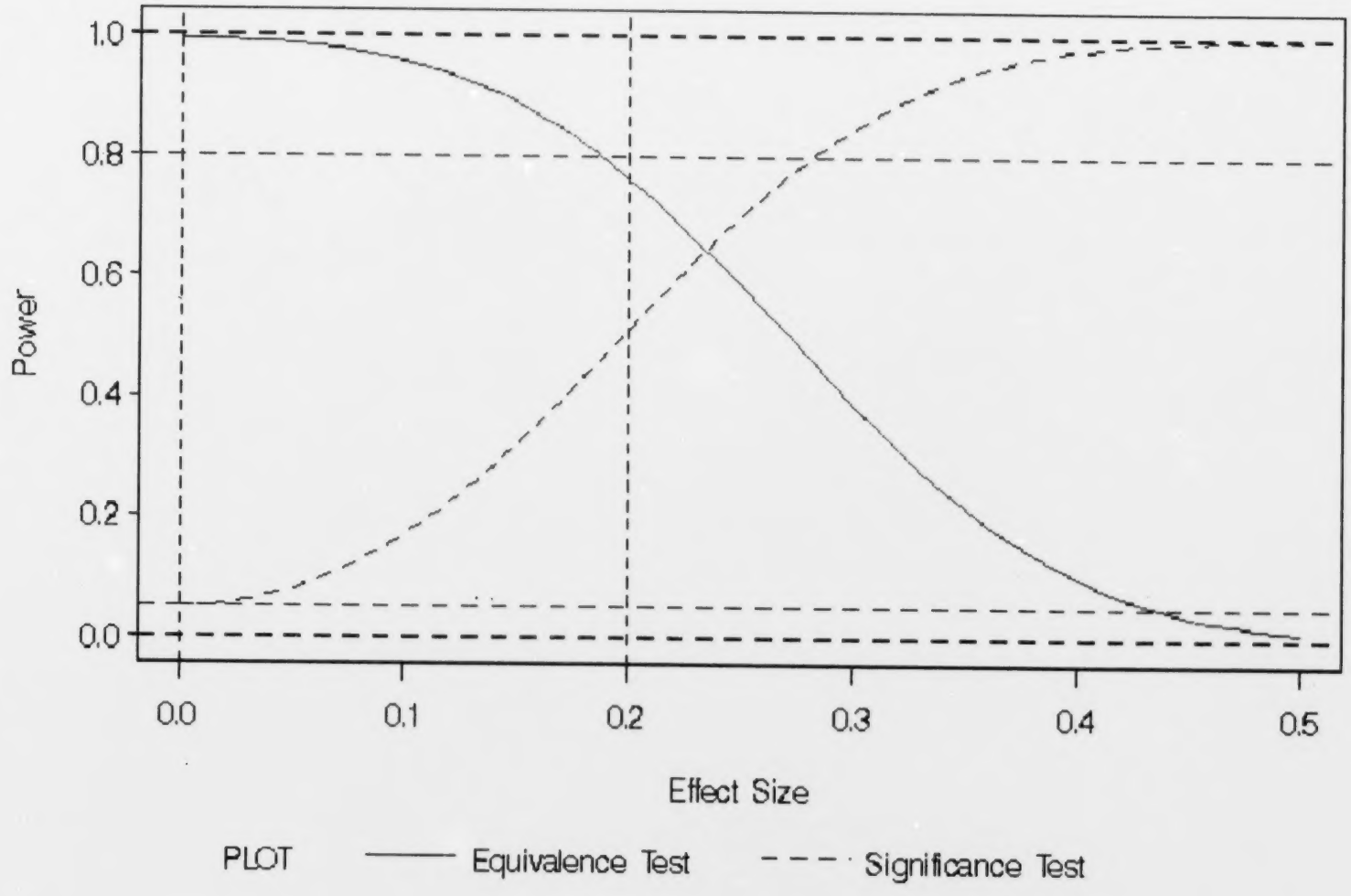
Figure 5. Power of the Tests of Significance and Equivalence with $\delta = 0.2$ and $\tau = 0.4$.

Figure 6. Simultaneous Power of the Tests of Significance and Equivalence with $\delta = 0$.

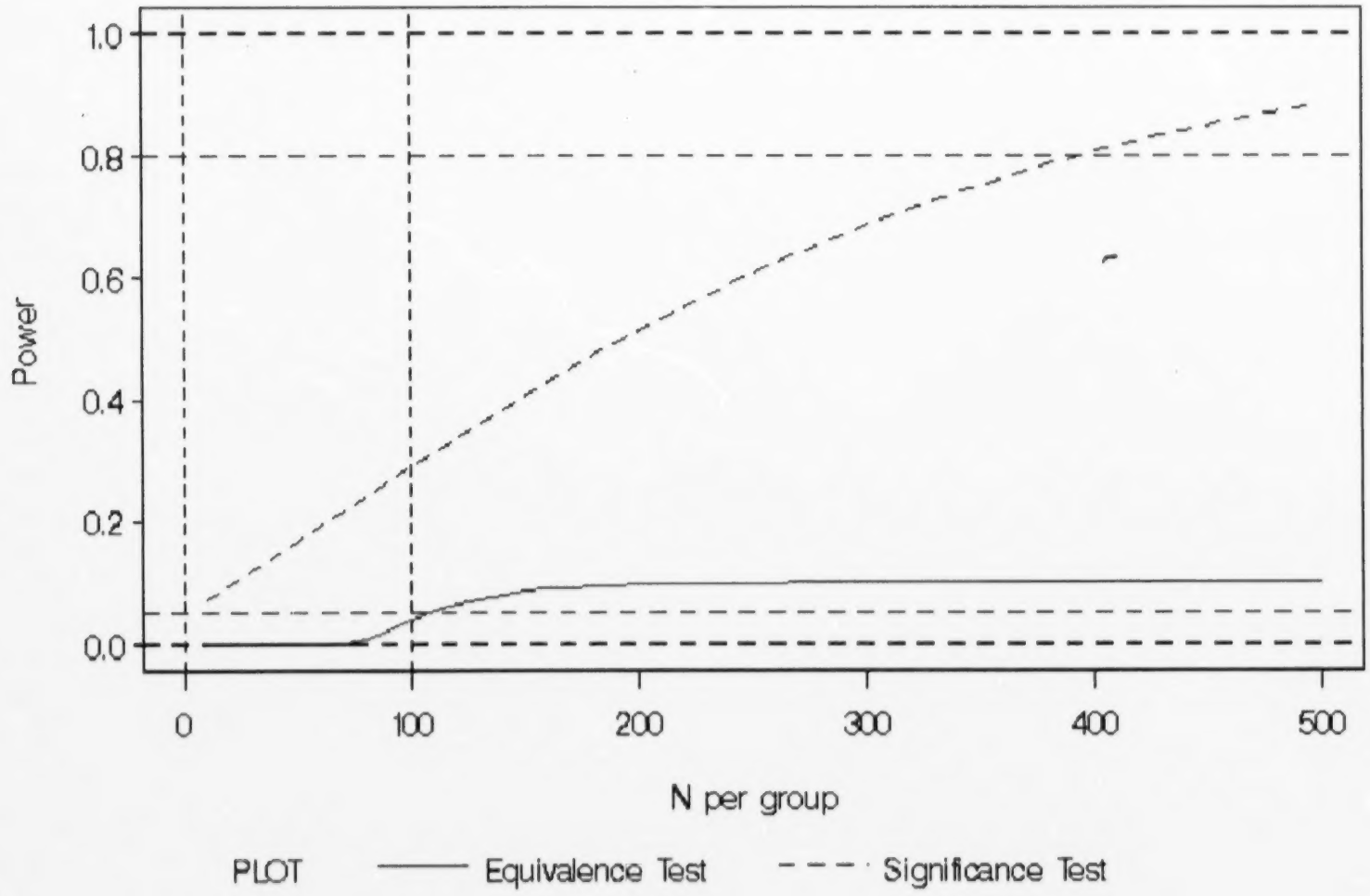
Figure 7. Simultaneous Power of the Tests of Significance and Equivalence with $\delta = 0.2$.



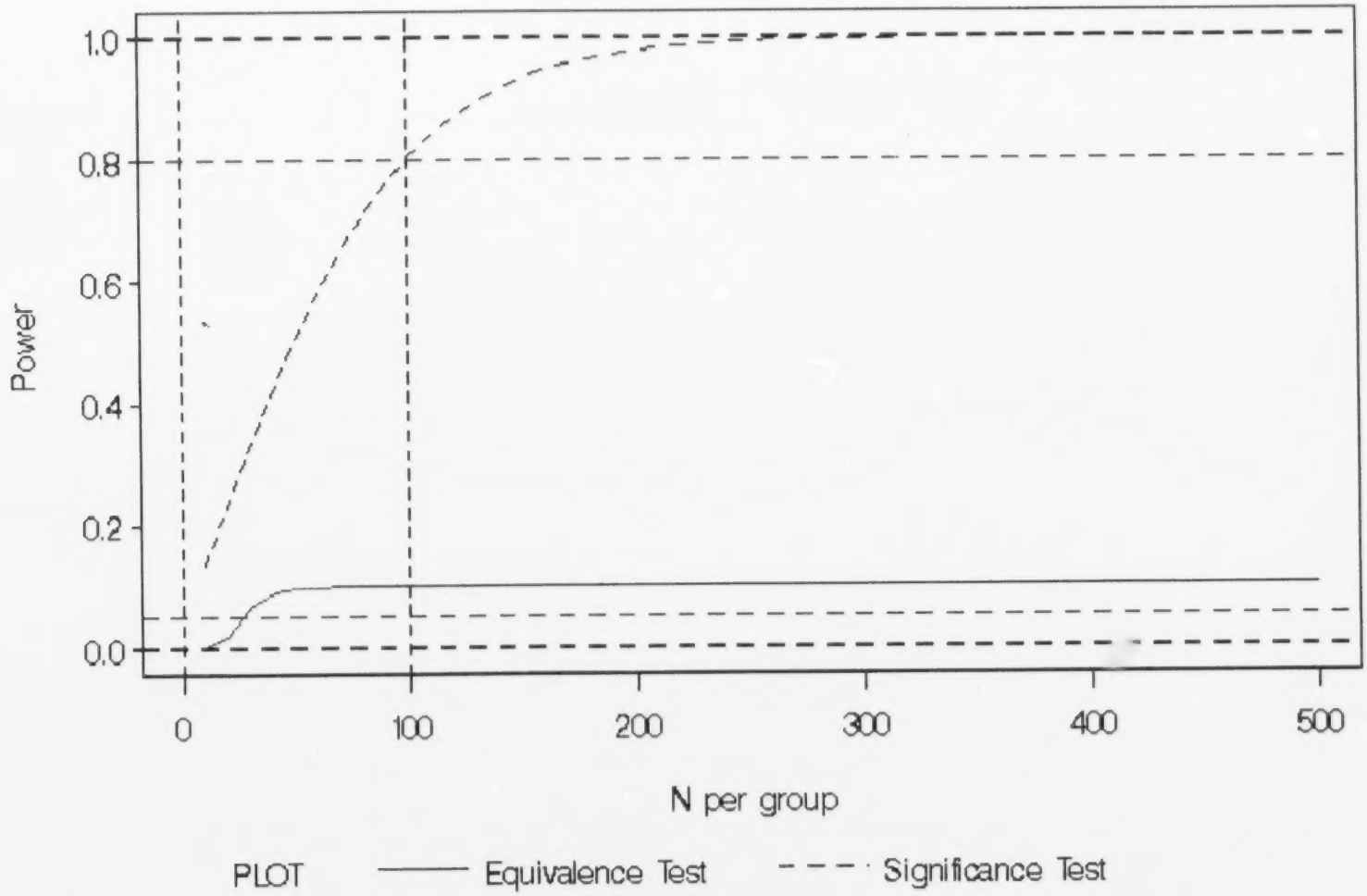
Equivalence Testing with Hypothesis Testing, Figure 1



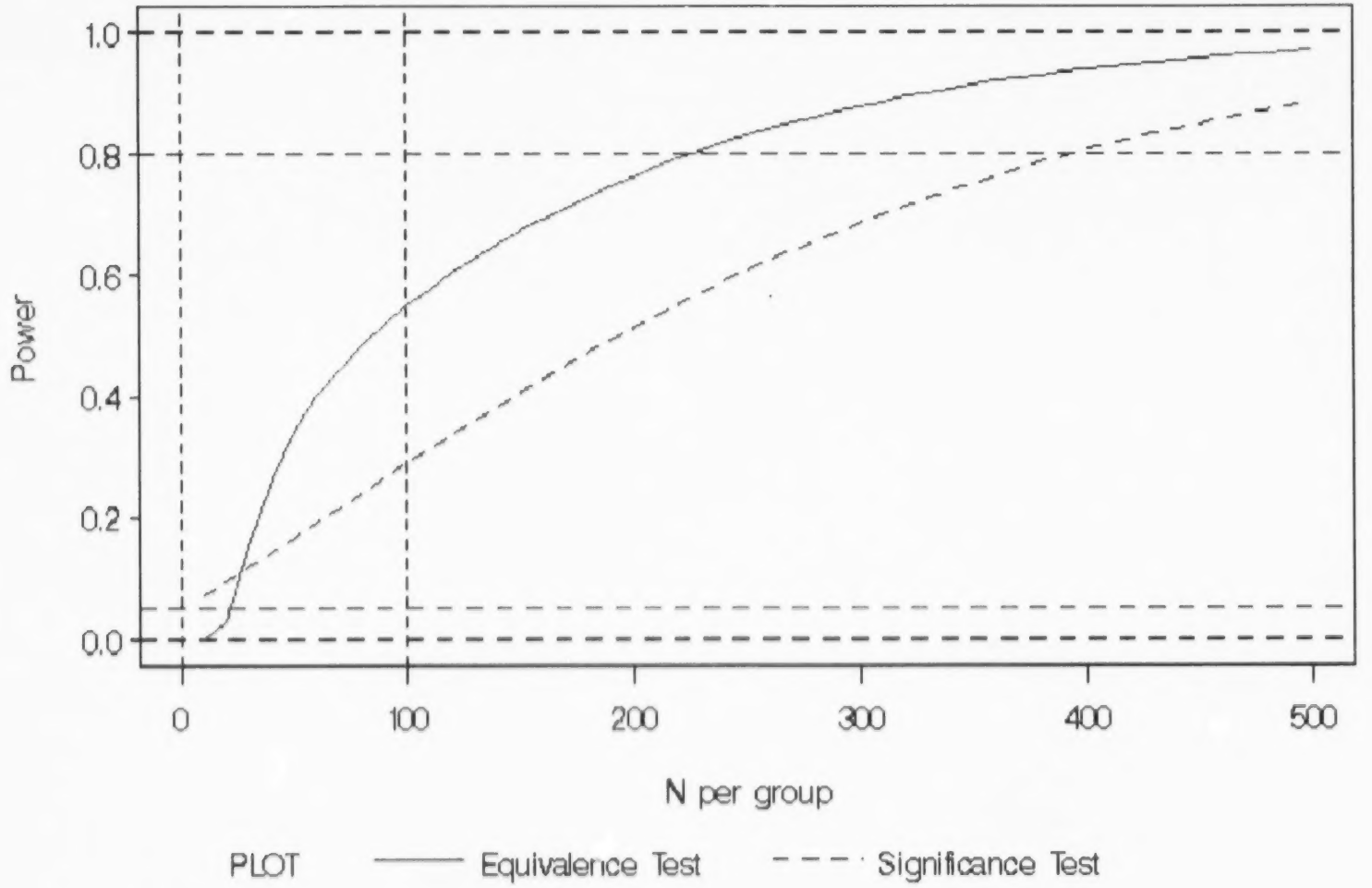
Equivalence Testing with Hypothesis Testing, Figure 2



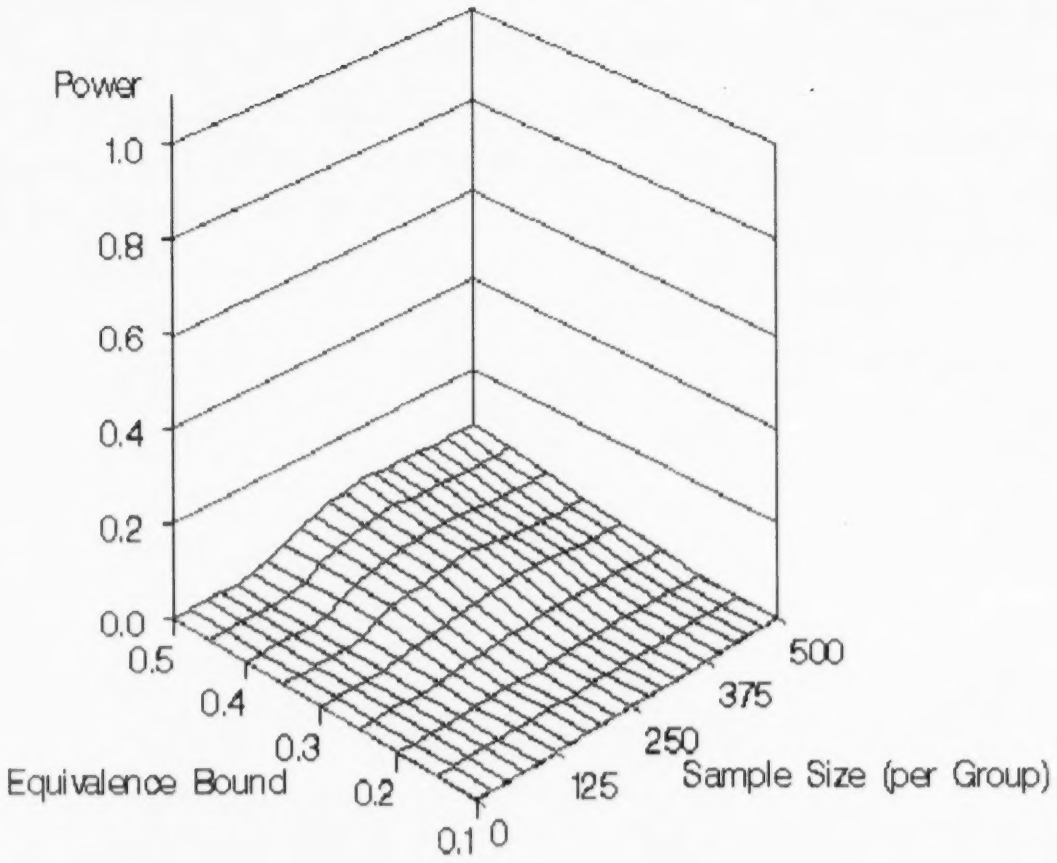
Equivalence Testing with Hypothesis Testing, Figure 3



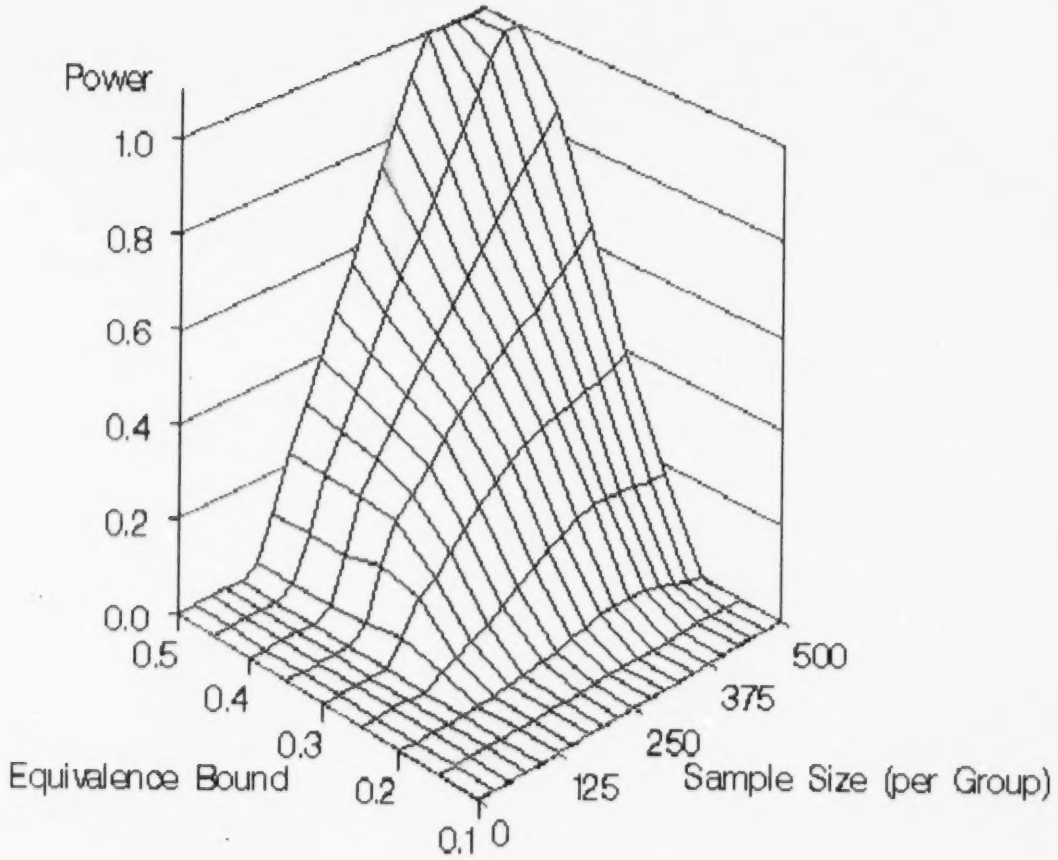
Equivalence Testing with Hypothesis Testing, Figure 4



Equivalence Testing with Hypothesis Testing, Figure 5



Equivalence Testing with Hypothesis Testing, Figure 6



Equivalence Testing with Hypothesis Testing, Figure 7

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